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Time Periodic Solution of the Burgers Equation on the Half-Line with Application to Flow Over Wavy Boundary

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Abstract

The evolution of large amplitude Tollmien-Schlichting waves in boundary layer flows over wavy surfaces is considered for two-dimensional disturbances which are locally periodic in time and space. Consideration is given to both large Re $\gg 1$ and Re $\sim O(1)$ Reynolds numbers using asymptotic methods. The large Reynolds number analysis is valid for oscillatory two-dimensional turbulent boundary layer. In both cases the phase equation approach shows that the wavenumber and frequency will develop shocks or other discontinuities as the disturbance evolves. It is shown that the evolution of constant frequency/wavenumber disturbances and their modulational instability is controlled by Burgers equation at finite Reynolds number and by a new integro-differential evolution equation at large Reynolds numbers. The Burgers equation is formulated on the half line, using Fokas' method, which provides a simple model of the above phenomenon. The physical situation corresponds to the solution of the Dirichlet problem on the half-line, which decays as $x \to \infty$ and which is *time periodic*. It is shown that the Dirichlet problem, where the usual prescription of the initial condition is now replaced by the requirement of the time periodicity, yields a well posed problem. Furthermore, it is also shown that the solution of this problem tends to the "inner" and "outer" solutions obtained by the perturbation expansions. For the large Reynolds number case the evolution equation points to the development of a spatially localized singularity at a finite time. The three-dimensional generalizations of the evolution equations is also given for the case of weak spanwise modulations.